ISSN 1022-4734

JUJSS
Vol. 34, 2017, pp. 59-68

Comparison of Constrained and Unconstrained Stochastic Frontier Production Function in Terms of Efficiency of the Estimates and Powers of the Tests-A Monte Carlo Simulation Approach

Md. Asraful Alam
Assistant Professor, Department of Statistics, Jahangirnagar University

Farhana Afrin Duty
Lecturer, Department of Statistics, Jahangirnagar University

and Ajit Kumar Majumder
Professor, Department of Statistics, Jahangirnagar University

Abstract

In most of the econometric problems as well as in stochastic frontier production function, the signs of the parameters are usually known in advance or may be constrained. If this type of prior information is included in the model, then such additional information may be valuable in increasing the efficiency of estimates and the probability of making correct decisions will be higher. As a consequence, the powers of the tests based on the constrained parameters are likely to be increased. Although there have been many studies about the stochastic frontier production function without restriction of the parameters, there are very few studies at all with restricted parameters of the same. So, our main aim is to develop the methods for estimating the parameters of the constrained stochastic frontier production function as well as to test them under restricted alternatives and to compare the constrained stochastic frontier production function with the unconstrained model in terms of efficiency of the estimates and the powers of the tests based on the estimates by Monte Carlo simulation. The results suggest that the estimates obtained from the constrained stochastic frontier production function are more efficient than those of the unconstrained estimates. By Monte Carlo simulation approach, it is also found that the powers of the tests based on the constrained parameters are higher than those of the unconstrained parameters. So, we can say that the constrained stochastic frontier production function is better than the existing model in terms of efficiency of the estimates and powers of the tests.

Key words: *Technical efficiency, stochastic frontier, constrained model, power.*

1. Introduction

In general forms of production functions, it is assumed that all the firms are technically efficient and the technological knowledge remains constant, that is there is no technological change in the production process. But in practical situations, all the firms are not technically efficient and the production scale can change also with the change in technology. Thus, these two assumptions of the production functions are unrealistic. When it is considered that all the firms are not technically efficient, the production frontier can be used to estimate the technical efficiencies among the firms.

The production frontier represents the optimum overall output attainable from the combined effect of each input level. Some inputs have negative effects on the overall output while some other inputs have positive effects. In most of the cases, these negative or positive effects of the inputs are known in advance. For example, an econometrician may know from theoretical arguments that the marginal propensity to consume lies between zero and one. Or it is known from past experience that if anyone expends his maximum of the times in study, his knowledge will increase.

If this type of prior information is included in the model, then such additional information may be valuable in increasing the efficiency of estimates and the probability of making correct decisions will be higher. As a consequence, the powers of the tests based on the constrained parameters are likely to be increased. If this type of prior information is incorporated in the model, then the efficiency of estimates and the powers of the tests are likely to be increased.

There have been many studies about the unconstrained stochastic frontier production function, but there is no study at all with restricted parameters of the same. The frontier version 4.1 can be used to estimate and test the parameters of the existing unrestricted stochastic frontier production function. But this program is not suitable for estimating and testing the parameters of the stochastic frontier production function when some of the parameters are restricted. For this reason, we have to write some sophisticated computer programs to estimate the parameters as well as to test the parameters of the constrained model and to compare the constrained model with the unconstrained model in terms of efficiency of the estimates and the powers of the tests based on the estimates by Monte Carlo simulation.

2. Objectives of the Study

1) To estimate the parameters of the stochastic frontier production function when some of the parameters are restricted.
2) To test the parameters of the stochastic frontier production function when some of the parameters are restricted.
3) To test whether there is any technical inefficiency effect in the stochastic frontier production function when some of the parameters are restricted.
4) To compare the efficiencies of the estimated parameters of the stochastic frontier production function when some of the parameters are restricted with respect to that of the unrestricted parameters.

5) To compare the powers of the tests based on the constrained parameters of restricted stochastic frontier production function with that of the unrestricted stochastic frontier production function by Monte Carlo Simulation approach.

3. Methodology

For the purpose of simulation, the data is taken from a survey conducted by the authors in July 2007 on the students of Jahangirnagar University. The total number of students is 3277. The sample size is 300. The detailed description of the determination of the size of the sample is given by Alam, Rois and Majumder (2011). The dependent variable is students' scores (%) and the independent variables are: spending time per day in formal study, self study, private tuition, reading newspapers, watching TV, mobile phone, leisure and sleeping (in minute).

4. Estimation of Constrained and Unconstrained Stochastic Frontier Production Function

Aigner, Lovell and Schmidt (1977) and Meeusen and Van Den Broeck (1977); Battese and Coelli (2005) independently proposed the usual unconstrained stochastic frontier production function, its estimation and test procedures. But we propose the constrained stochastic frontier production function as follows:

$$\ln(Y_i) = X_i\beta + v_i - u_i \quad ; \quad i = 1, 2, \ldots, n$$

Subject to the constraints: $\beta \in C$, where $\beta \in R^P$ is a sub-vector of unknown parameter space $\Theta \in R^S$ and C is a subset of R^P. For our data, the constrained stochastic frontier production function may be written as:

$$\ln(Y_i) = \beta_1 + \beta_2 \ln\left(X_{2i}\right) + \ldots + \beta_9 \ln\left(X_{9i}\right) + v_i - u_i \quad ; \quad i = 1, 2, \ldots, 300$$

Subject to the constraints: $\beta_2 > 0$, $\beta_3 > 0$, $\beta_9 > 0$

The parameters of the constrained stochastic frontier production function can be estimated using the maximum likelihood (ML) method, which requires numerical maximization of the following likelihood function:

$$\ln(L) = -\frac{n}{2}\ln\left(\frac{\pi}{2}\right) - \frac{n}{2}\ln\left(\sigma_s^2\right) + \sum_{i=1}^{n}\ln\left(1-\Phi\left(z_i\right)\right) - \frac{1}{2\sigma_s^2}\sum_{i=1}^{n}\left(\ln\left(Y_i\right) - X_i\beta\right)^2$$

Subject to the constraints: $\beta \in C$,

where $\sigma_s^2 = \sigma_u^2 + \sigma_v^2$, $\gamma = \frac{\sigma_u^2}{\sigma_s^2}$, $z_i = \frac{\ln\left(Y_i\right) - X_i\beta}{\sigma_s}\sqrt{\frac{\gamma}{1-\gamma}}$ and $\Phi(\bullet) =$ distribution function of the standard normal variable.

The maximum likelihood (ML) estimates of β, σ_s^2 and γ are obtained by finding the maximum of the above log-likelihood function. These estimators are consistent and asymptotically efficient (Aigner, Lovell and Schmidt (1977). Frontier version 4.1 is unable to estimate the parameters of this type of model when some of the parameters are restricted. We have to write program to obtain the optimum estimates of the parameters from the above likelihood function. This program uses a three-step estimation procedure, which is given bellow:

1) The first step involves calculation of the ordinary least squares (OLS) estimators of β and σ_s^2.
2) In the second step, the likelihood function is evaluated for a number of values of γ between zero and one. In these calculations, the OLS estimates of σ_s^2 and β_1 are adjusted by:

   $\sigma_s^2 = \sigma_{OLS}^2 \frac{\pi(n-k)}{n(\pi - 2\hat{\gamma})}$ and $\beta_1 = \beta_{1(OLS)} + \sqrt{\frac{2\hat{\gamma}\hat{\sigma}_s^2}{\pi}}$, respectively.

   The OLS estimates are used for the remaining parameters in β.
3) The final step uses the best estimates from the second step as starting values in a Davidon-Fletcher-Powel (DFP) iterative maximization routine, which obtains the maximum likelihood (ML) estimates when the likelihood function attains its global maximum.

Approximate standard errors of the maximum likelihood (ML) estimators are calculated by obtaining square roots of the diagonal elements of the direction matrix from the final iteration of the DFP routine. The direction matrix from the final iteration is usually a good approximation for the inverse of the Hessian of the log-likelihood function, unless the DFP routine terminates after only a few iterations.

5. Tests of Hypotheses

For the frontier model, the null hypothesis that there are no technical inefficiency effects in the model can be conducted by testing the null and alternative hypotheses as follows:

$$H_0: \gamma = \frac{\sigma_u^2}{\sigma_s^2} = 0 \quad \text{vs} \quad H_A: \gamma = \frac{\sigma_u^2}{\sigma_s^2} > 0$$

The above hypothesis can be tested by one-sided generalized likelihood ratio test. The test statistic is given by:

$$LR = -2\ \ln\left(\frac{L(H_0)}{L(H_A)}\right)$$

where, $L(H_0)$ and $L(H_A)$ are the values of the likelihood function under the null and alternative hypotheses, respectively.

Under the null hypothesis, this test statistic is assumed to be asymptotically distributed as a mixture of chi-square distribution with degrees of freedom equal to the number of restrictions involved (in this instance one), namely, $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$. At $\alpha\,\%$ level of significance, the critical value is $\chi_1^2\ (2\alpha)$.

It is to be noted here that the regular (two-sided) generalized likelihood ratio test was included in the Monte-Carlo experiment in Coelli (1995) and shown to have incorrect size (too small), as expected.

6. Results

The ordinary least squares (OLS) estimators of β and σ_s^2 are given in Table 1. Then, the likelihood function is evaluated for a number of values of γ between zero and one and the OLS estimates of σ_s^2 and β_1 are adjusted, which are also given in Table 1.

Table 1: The Ordinary Least Squares (OLS) Estimates and the Estimates after the Grid Search

The ordinary least squares estimates			**Estimates after grid search**
	Coefficients	t-ratio	Coefficients
Intercept	-0.281179016	-0.314482068	-0.175873
Formal study	0.649315288	12.58837782	0.6493153
Self-study	0.132279853	2.050283984	0.1322799
Private	0.004980624	1.336616669	0.0049806
News paper	0.017663028	1.354758783	0.017663
Watching TV	0.0159634	1.533599081	0.0159634
Mobile	-0.048075159	-3.075117435	-0.0480752
Leisure	-0.198329091	-3.804696329	-0.1983291
Sleeping	0.194899082	3.836246385	0.1948991
Sigma square	0.00881296		0.0196379
Gamma			0.8870143
log likelihood function = 281.2538			

The final estimates of the constrained stochastic frontier production function is obtained by using the estimates obtained after the grid search as starting values in a DFP iterative maximization routine. The final estimates of the restricted stochastic frontier production function are given in Table 2**.**

Table 2: Final Maximum Likelihood Estimates of the Constrained Stochastic Frontier

	Coefficients	**Standard error**	**t-ratio**
Intercept	-0.192406378	0.53740103	-0.358031279
Formal study	0.759098233	0.031848084	23.83497334*
Self-study	0.052308142	0.022897592	2.284438556*
Private	0.015742589	0.003120766	5.044463122*
News paper	0.004942164	0.010562806	0.467883629
Watching TV	0.00680172	0.009224692	0.737338439
Mobile	-0.053527507	0.014160521	-3.78005209*
Leisure	-0.199036032	0.036081816	-5.516242087*
Sleeping	0.172523658	0.033517231	5.147312378*
Sigma square	0.020786394		
Gamma	0.919251983		
log likelihood function = 302.5803			
LR test of the one-sided error = 42.653			
Mean technical efficiency = 0.90			

From the results obtained from Table 2, we found that the likelihood ratio test of the one-sided error (42.653) is greater than the tabulated value (2.71)**.** So, the null hypothesis of the absence of the technical inefficiency effect is rejected. So, we can say that the use of constrained stochastic frontier production function (from LR test) is appropriate. It is also seen that both formal study and self-study are significant determinants (* indicates that the parameters are significant) of examination scores but the former is more important than the latter. Times spent in mobile phone and leisure has negative effect on the performance of the students.

So, the students must have to spend their maximum of the times in formal lectures and classes than that in self-studies and should not spend their maximum of the times in mobile phone and in leisure. They should sleep every day for some period of time. This will help them to do all other works properly. The University authority and the government could encourage the teachers to take the classes seriously and also the students to attend in lectures and classes by seminars, symposiums, logistic supports etc. or even could make it compulsory.

Table 3: Efficiency of Constrained Estimators with Respect to Unconstrained Estimators

	Variances of the unconstrained estimates	**Variances of the constrained estimates**	**Efficiency of constrained estimates with respect to unconstrained estimates**
Intercept	0.07078032	0.070401	1.005392
Formal study	0.0000924	0.000092	1.004398
Self-study	2.544848	1.98233	1.283766
Private	0.008482025	0.008276	1.024912
News paper	0.01363089	0.010315	1.321523
Watching TV	0.0000441	0.0000406	1.085558
Mobile	0.000545487	0.000543	1.004561
Leisure	0.000382196	0.000374	1.022949
Sleeping	0.000824173	0.00082	1.004748

From Table 3, it is seen that the estimates obtained from the constrained stochastic frontier production function are more efficient than those of the unconstrained estimates. So, the stochastic frontier production function with restricted parameters is better than that of unrestricted parameters.

Now, by Monte Carlo simulation approach, the powers of the tests based on the constrained and unconstrained parameters are shown in the following figures:

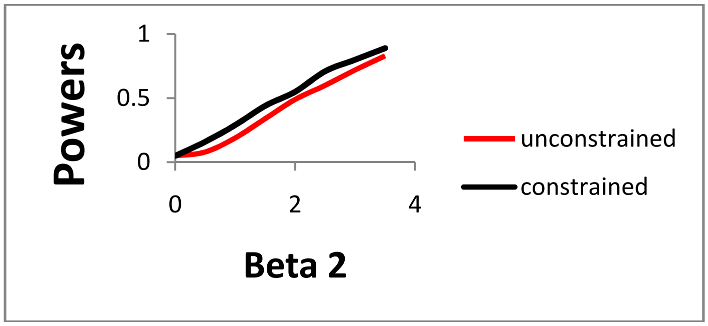

Figure 1. Power curves of the tests based on the constrained and unconstrained parameters for β_2.

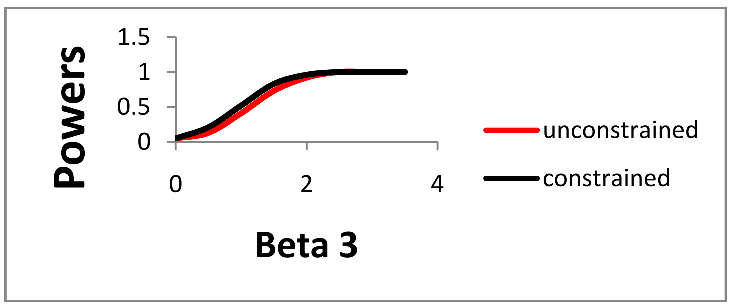

Figure 2. Power curves of the tests based on the constrained and unconstrained parameters for β_3.

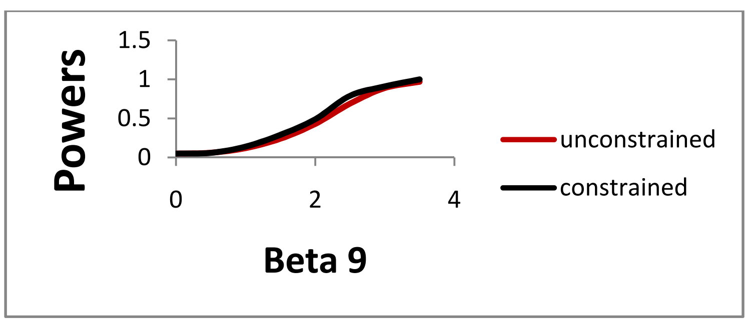

Figure 3. Power curves of the tests based on the constrained and unconstrained parameters for β_9.

From the above three figures, it is seen that the powers of the tests based on the constrained parameters are higher than those of the unconstrained parameters. So, the constrained stochastic frontier production function is better than the unconstrained model in terms of power properties.

7. Conclusion

We have developed the estimation technique and testing procedure of the restricted stochastic frontier production function. It is found that the estimates obtained from the constrained stochastic frontier production function are more efficient than those of the unconstrained estimates. By Monte Carlo simulation approach, it is also found that the powers of the tests based on the constrained parameters are higher than those of the unconstrained parameters. So, we can say that the constrained stochastic frontier production function is better than the unconstrained model in terms of efficiency of the estimates and powers of the tests.

References

A. Alam, R. Rois, and A. K. Majumder (2011): "Measuring the factors of examination performance-a stochastic frontier production function: case study on Jahangirnagar University", *ASA University Review*, Vol. 5 No. 2 (9th Issue, July-December), pp 47-57.

D. J. Aigner, C. A. Lovell, and P. Schmidt (1977): "Formulation and estimation of stochastic frontier production function models", *Journal of Econometrics*, 6(1), pp 21–37.

G. Battese and T. Coelli (1988): "Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data", *Journal of Econometrics,* 38, pp 387–399.

G. Battese and G. Corra, (1977): "Estimation of a production frontier model: with application to the Pastoral zone of eastern Australia", *Australian Journal of Agricultural Economics*, 21, pp 167–179.

G. S. Becker (1965): "A theory of the allocation of time", *The Economic Journal*, 75 (299), pp 493–517.

J. Bound, D. Jaeger and R. Baker (1995): "Problems with instrumental variables estimation when the correlation between the instruments and the endogenous explanatory variable is weak", *Journal of the American Statistical Association*, 90, 430.

T. J. Coelli, D. S. P. Rao, C. J. O'Donnel and G. E. Battese (2005): *An Introduction to Efficiency & Productivity Analysis,* Springer.

E. Cohn and S. Cohn (2000): "Class attendance and performance in Principles of Economics", *Proceedings of the Business and Economic Statistics Section*, American Statistical Association, pp 201–206.

G. C. Durden and L. V. Ellis (1995): "The effects of attendance on student learning in Principles of Economics", *American Economic Review*, 85(2), pp 343–346.

W. Greene (1990): "A gamma distributed stochastic frontier model", *Journal of Econometrics*, 46, 141–163.

F. Juster and F. Stafford (1986): "Response errors in the measurement of time use", *Journal of the American Statistical Association*, 81(394), pp 390–402.

F. Juster and F. Stafford (1991): "The allocation of time: empirical findings, behavioral models, and problems of measurement", *Journal of Economic Literature*, *XXIX*(June), pp 471–522.

M. Hossain, A. Alam, and K. Uddin (2015): "Application of Stochastic Frontier Production Function on Small Banana Growers of Kushtia District in Bangladesh", *Journal of Statistics Applications & Probability,* Volume 4, No. 2, pp 337-342.

W. Meeusen and J. Van Den Broeck (1977): "Efficiency estimation from Cobb–Douglas production functions with composed error", *International Economic Review*, 18, pp 435–444.

C. Mulligan, B. Schneider and R. Wolfe (2000): "Time use and population representation in the Sloan study of adolescents", *NBER Technical Working Paper,* 265.

M. Norman and B. Stocker (1991): *Data envelopment analysis,* New York, John Wiley and Sons.

Schmidt, R. M. (1983), "Who maximizes what? A study in student time allocation" *American Economic Review*, 73(2), pp 23–28 papers and proceedings.

S. P. Singh, A. K. Parashar and H. P. Singh (1999): *Econometrics and Mathematical Economics,* S. Chand and Company Ltd.