

Determining Probability of Extinction of the Traditional Handloom Industry in Bangladesh

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Abstract

Handloom industry is one of the ancient and traditional small household based industries in Bangladesh. Generally, the skill of handloom weaving passes through inheritance among generations. In this paper we assess appropriate model for determining generation size of handloom weavers as well as the probability of extinction of this profession. The data is collected through a survey in Sujanagar, Pabna. We apply branching process to fit model and find the probability of extinction. The study reveals that branching process with immigration is a suitable choice for this situation. The offspring distribution follows geometric pattern and the process has a high probability of extinction if there is no immigration. The immigration can either delay the extinction or make the process to converge to a limiting distribution in the long run. We also show simulated results using different immigration pattern.

Keywords: Branching process with immigration, extinction probability, offspring distribution.

1. Introduction

The handloom industry is the ancient, the biggest and the most important household based industry in Bangladesh. This is not just a cloth material or traditional wear; it symbolizes to the Bangladeshi civilization, which is one of the oldest on the planet. We had a long tradition of excellence in making high quality of handloom products with extraordinary skills and craftsmanship when no such skills were available anywhere in the world. The handloom sector is not only inherits our past glory but also plays a vital role even in the context of present Bangladeshi economy. The level of artistry and intricacy achieved in the handloom fabrics is unparalleled and beyond the scope of modern machinery. It represents diverse cultural ethos ranging from exquisite fabrics, which take months to weave, to popular items of mass production for daily use. Hand weaving is one of the richest and the most resilient media of ethnic expression.

The handloom sector in Bangladesh consists of more than 0.183 million handloom units with 0.505 million handlooms and about 1 million handloom weavers (Bangladesh Handloom Board). A manpower of about one million weavers, dyers, hand spinners, embroiderers and allied artisans have been using their creative skills into more than 0.30 million active looms to produce around 687 million meters of fabrics annually (Bangladesh Handloom Board). Production of these handloom fabrics is diffused in numerous production centers all over the country which are linked up by a network of primary, secondary and central markets.

It is largely home-based; with labor inputs form the entire family. While weaving sheds do exist occasionally, more widespread is the weaver weaving at home, drawing on the labor of all the family

members. In each stage of the production process, whether it is pre-loom processing, weaving or finishing – every member of the household has a clear role to play. In many ways, it is similar to the subsistence agricultural household that engages in the self-exploitation of labor in order to remain at the subsistence level (Chowdhury, 1989).

But unfortunately, this ancient and one of the most important cottage industries of Bangladesh now seems on the way of extinction because of various problems and barriers attached to this industry. Gosh (2005) has identified several barriers that are responsible for its extinction which include shortage of working capital, high cost of raw material procurement, lack of organizing capability, inadequate technology and efficiency, and lack of policy support are major forces which bitterly hit the handloom industry. The major competitors of handloom products are (1) cloths come through legal and illegal ways from other countries and 2) power loom produced cloths (Ahmed, 1999). The share of the cloth production in the handloom sector dropped sharply while the share of cloth production in mills jumped up tremendously. So it is necessary to take urgent steps in reviving the sector.

In this study, we want to model the generation of weavers. Since most of the cases, the knowledge and skills needed for the handloom sector usually transfers from their forefathers, we can model the generation of the handloom weaver by using branching process. This study may help the Government and Non-Government organizations in taking necessary steps to revive the handloom weaving in Bangladesh. Under this backdrop the objective of the study is to model the generation size of weavers using branching process and determine the probability of extinction of this industry.

The next section will discuss the methodology of this study which includes theoretical background of branching process, data collection methods and limitations of this study. Section 3 is devoted to the results and discussion and in Section 4 we make the concluding remarks.

2. Methodology

Branching processes was first modeled mathematically Galton and Watson (1874). They had formulated the model for the ‘extinction of families’. Though the model did not attract much attention for a long time; the situation gradually changed and during the last 50 years as much attention has been devoted to it Medhi (1982). Now-a-days branching process is used in many areas of research.

A branching process is a mathematical description of the growth of a population where the individual produces offspring according to stochastic laws. A typical process is of the following form. Considering a population of individuals developing from a single progenitor - the initial individual produces a random number of offspring, each of them in turn produces a random number of offspring; and so the process continues as long as there live individuals in the population. An interesting question is the probability that the population survives, (or, extinct).

Let us assume that an individual (which can be an atomic particle, or a tsetse fly or a mosquito) produces a random number ξ of offspring, and let

$$(\xi = k) = p_k, k=0,1,2, \quad (2.1)$$

be the probability mass function for the number of offspring generated by each individual. Individuals give rise to offspring independently so ξ is identically and independently distributed for each individual within a generation time n . Since the individuals behave independently, then the size of each generation $\{X_n, n \geq 0\}$ follows a discrete-time branching process.

In the n^{th} generation, the X_n individuals independently produce other individuals $\xi_1^{(n)}, \xi_2^{(n)}, \dots, \xi_n^{(n)}$ and thus the total number of individuals produced in the $(n+1)$ th generation is

$$X_{n+1} = \xi_1^{(n)} + \xi_2^{(n)} + \dots + \xi_n^{(n)} \quad (2.2)$$

This is the fundamental branching process equation. However, in probability theory, a branching process is a Markov process that models a population in which each individual in generation n produces some random number of individuals in generation $n+1$, according to a fixed probability distribution that does not vary from individual to individual. The branching processes are used to model reproduction; for example, the individuals might correspond to bacteria, each of which generates 0, 1, or 2 offspring with some probability in a single time unit.

We have proposed branching process with immigration for modeling the job changing behavior and probability of extinction among the weavers. The following branching process with immigration is used in the study:

$$X_{n+1} = \sum_{r=0}^{X_n} \zeta_r + Y_{n+1}$$

Here,

X_{n+1} denotes the $(n+1)$ th generation of the process,

ζ_r denotes the number of offspring from the r^{th} individual of n^{th} generation and

Y_{n+1} indicates the number of immigrants into the $(n+1)$ th generation.

We can consider offspring of an individual as who take the same profession as one's ancestor. This leads to a similar problem that Galton-Watson investigated, that is the probability of extinction of family names. However, in our case there is some new people come in the profession. We treat this type as immigration and use a model branching process with immigration. So, we propose to use branching process with immigration to model the generation size and calculating the probability of extinction among the handloom weavers. Naturally this model can be extended to other profession as well. We use the following assumptions:

- If the son/daughter of a weaver comes into the profession of the parent it is considered as an offspring. The frequency of offspring was calculated from the survey data and fit into a suitable distribution.
- If a person comes into the profession from another profession, we consider it as immigration.

The data has been collected purposively from the weavers at Sujanagar Thana in Pabna through a well-designed questionnaire. Data was collected by the direct interview method while the total number of interviewed weavers is 195.

To get necessary information from the data the survey ask questions on how many of the sons of a particular person is also a handloom weaver by profession. This helps to find the offspring distribution. Another important question is whether the person is a handloom weaver by heredity or he is the first person in the profession. This information was used to determine the new persons in the profession which will be used in the model.

The study is quite limited in the sense that, it uses a small sample from a particular area. The other thing is that the sample is from one time period. The historical time series data for a particular area would have enriched the estimation and help to find more specific distribution pattern for both offspring and immigration. But the related survey misses this type of information. We recommend to include questions about coming to this profession, father’s profession, grandfather’s profession and number of child taking the same profession in future surveys of such kind. This will help to determine the pattern of population size. This will also help to develop and test new models for this.

3. Result Discussion

3.1 Offspring distribution

During the survey, questions were asked on how many of the sons/daughters of a weaver also joined the profession. Table 1 shows the results we get (see figure 1 for a graphical representation).

In this study, three discrete distributions for this data namely geometric, Poisson and negative binomial distributions have been used. Figure 2 shows the fitness of each of the distribution on the data. For Poisson distribution the best fitted model has the $\lambda = 0.46$, for geometric distribution the best fit occurs with $p = 0.68$ while the negative binomial distribution has size 1 and $p = 0.75$. To select the best model a chi-squared goodness of fit test is conducted over the models. Table 2 summarizes the result of the chi-square tests and indicates that the geometric distribution with $p = 0.68$ is found to be the best fit among the three.

Table 1: Frequency distribution of number of offspring

Number of offspring	Frequency
0	113
1	30
2	18
3	2
4	1

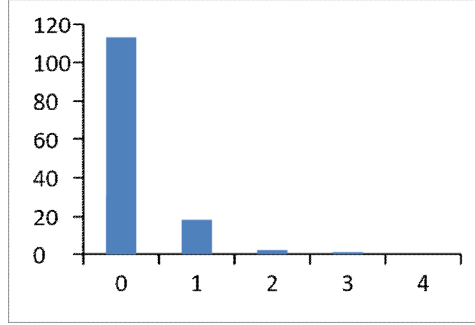


Figure 1: Histogram of the number of offspring

Table 2: Goodness of fit of different distribution

Distribution	Parameters	χ^2	Degrees of freedom	p - value
Poisson	$\lambda = 0.46$	14.74	4	0.005
Geometric	$p = 0.68$	5.86	4	0.209
Negative Binomial	$n = 1, p = 0.75$	16.02	4	0.029

So, the offspring distribution is found to be geometric ($\zeta \sim \text{geometric}(0.683)$) which gives the mean and variance of the distribution.

$$E[\zeta] = \frac{1-p}{p} = \frac{1-0.68}{0.68} = 0.46$$

$$VAR[\zeta] = \sigma^2 = \frac{1-p}{p} = \frac{1-0.68}{0.68^2} = 0.68$$

By using the results above the mean and variance of generation size X_n can be obtained as:

$$E[X_n] = (E[\zeta])^n = (0.46)^n$$

$$Var[X_n] = \sigma^2 \mu^{n-1} \left(\frac{1-\mu^n}{1-\mu} \right) = 0.68 \times 0.46^n \left(\frac{1-0.46^n}{0.53} \right)$$

The mean of offspring distribution is less than 1 indicating that the branching process is a subcritical process. So the process is guaranteed to extinct. We can find the generating function of the offspring distribution is given by:

$$G(s) = \frac{0.68}{1-0.32s}$$

and we can get the expected time of extinction is found to be

$$E[T] = \frac{1}{1-p} = 3.15$$

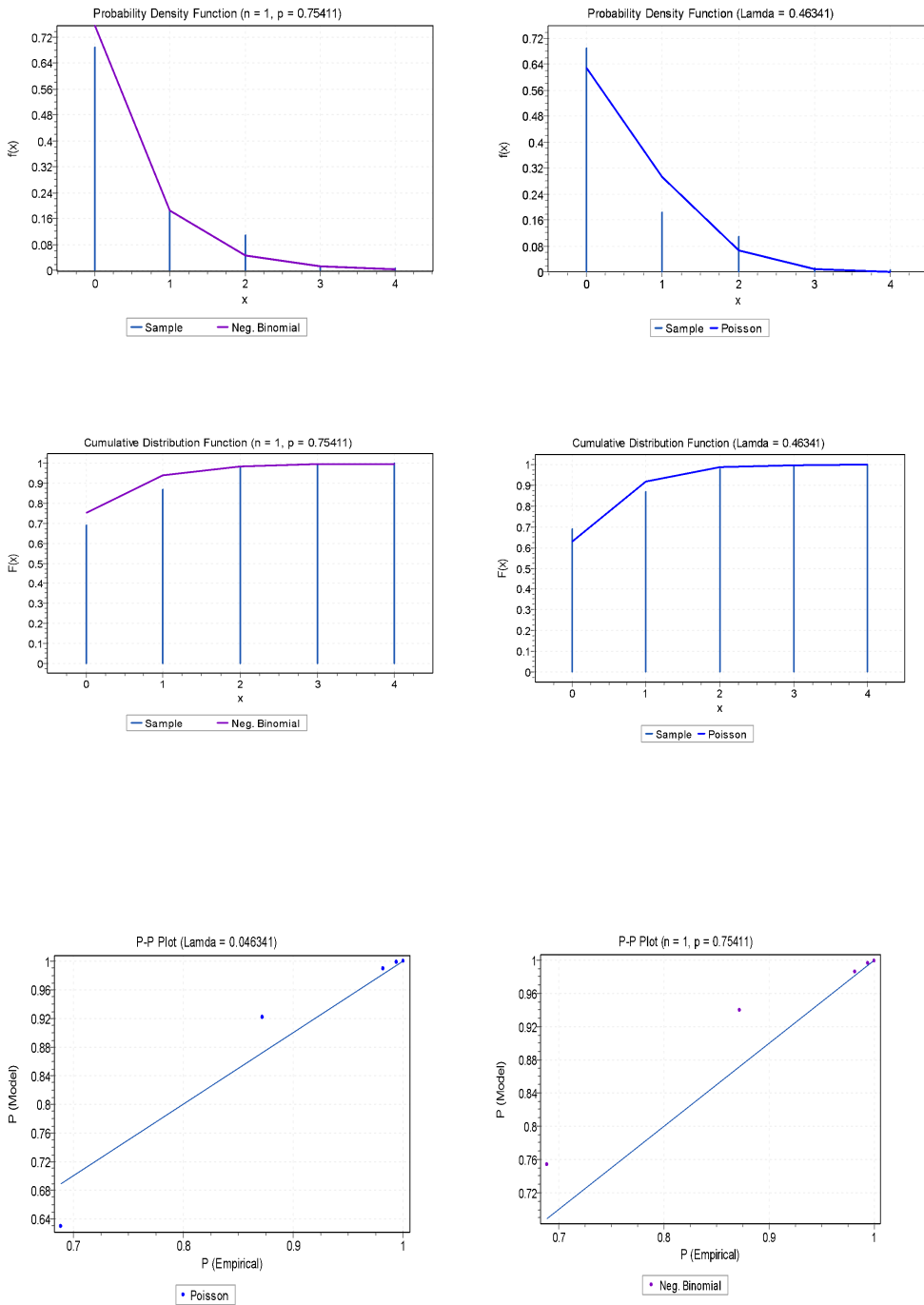


Figure 2: Fitting Geometric, Negative Binomial and Poisson distribution

So, in general case the process is expected to die out in 3-4 generations. However, this is not the case. The handloom industry is still running for years and generations. Our survey suggests that this is because of immigration. We have not considered immigration yet. This will give us better insight over the process.

3.2 Effect of immigration

At this point, we get that branching process cannot model the weavers' generation alone. If it would be the case, the weaver' generation would have been extinct by now. But this is not the original scenario. The reason behind this is the number of persons those come into the profession from other types of jobs that is immigration. Now the available data seem to be insufficient, in building a model for immigration data (table 2). Also no other available sources are found from where we may have the necessary information.

Table 3: Proportion of immigration in different generation

Sample Size	Immigration size	Percentage
72	13	18.1
52	11	21.2
81	15	18.5

However, there may be two effects of immigration. It may delay the extinction or it may lead the process to a limiting distribution. To check such effects we use simulation studies with immigration data with different distributions.

The study considers an initial population of 20000 and the process has the geometric offspring distribution. Then using generation sizes for the next 25 generations are conducted. Simulation study runs for 1000 times and also calculates the probability of being extinct within the next 25 generations.

3.2.1 Simulation 1

In this simulation, we assume that the immigration depends on the previous generation, that is a proportion of previous generation and this proportion comes from a uniform distribution from [0.0, 0.5]. In this case, we see that the extinction is delayed and the population size decreases. For this the probability of extinction is low at 10th generation but it is almost certain at 20th generation.

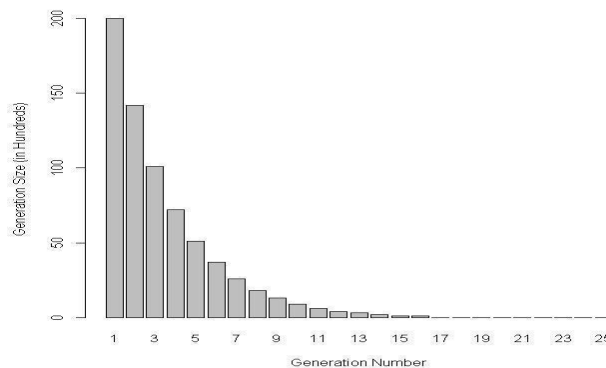


Figure 3: Generation Size for Simulation 1 (uniform proportion)

3.2.2 Simulation 2

In this simulation, the immigration from a Poisson distribution with $\lambda = 40$ is conducted. This simulation study gives an interesting result. The study reveals that the process does not extinct even at the 25th generation. However, after a few generations the generation size becomes stable and continues to maintain that. So the immigration from a Poisson distribution leads to a convergence instead of extinction.

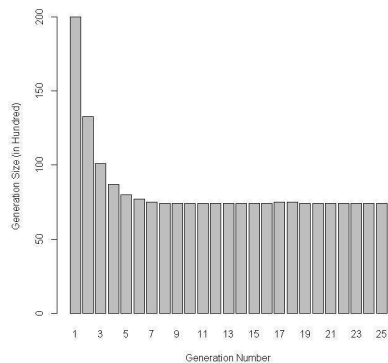


Figure 4: Generation Size for Simulation 2 (Poisson)



Figure 5: Generation Size for Simulation 2 (Geometric)

3.2.3 Simulation 3

In this case, simulation has been done considering the immigration from a geometric distribution with $p = 0.5$. With this case the extinction occurs but it is slower than simulation 1. This study observes a low probability of getting extinct within 10th generation but it increases to 40% within 20th generation and up to around 50% within 25th generation. Figure 3 shows the generation sizes for the Simulation 1, Simulation 2 and Simulation 3. (Merge figure 3, 4 and 5)

Table 4: Summary of the simulation results

Immigration Distribution	Mean	Variance	Probability of extinction within 10 generations	Probability of extinction within 20 generations	Probability of extinction within 25 generations
Uniform	28.04	2614.79	.065	0.926	0.987
Poisson	83.48	752.34	0.0	0.0	0.0
Geometric	28.72	1648.46	0.126	0.388	0.491

4. Conclusion

The study modeled the generation size of the weavers using branching process, estimated the offspring distributions and find the mean, variance and generating function of the process. The offspring distribution follows geometric distribution. The extinction probability of the process has been calculated. Further, we derived few simulation scenarios to understand the effect of immigration from different distributions.

The offspring distribution is observed to be subcritical and thus guaranteed to be extinct. The expected time for extinction is around four generation. But the process is going on for long time in the real world. This is because of new individuals come into the professions from other ones. It is considered as immigration and model the whole process as branching process with immigration. The immigration is delaying the extinction or taking the process to some limiting distribution. Unfortunately, data are limited to estimate the pattern of immigration. So simulate immigration from different distributions are conducted. Interestingly, from the simulation, we get similar result as we expected. If the immigration follows a uniform proportion distribution or geometric distribution extinction is delayed where immigration from Poisson distribution leads the process to converge to a limiting distribution.

We recommend that regular survey on weavers or any other professionals should include some questions on how they come to the profession and whether their fathers, grandfathers belong to the same profession. There should also be questions on how many of the sons/ daughters have taken their father's profession. Regular collection of such data will provide an excellent database to analyze such model. This will constitute to historical data help one analyzing the origin and start time of a profession. This will also help understanding the validity of modeling such generation sizes. The branching process with immigration provides a good framework for analyzing such situations.

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