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A Note on Modified Umbrella Test for Randomized Block Design with Tied Observations and Ordered Alternatives

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Abstract

This paper treats the problem of umbrella pattern treatment effects in a randomized block design when data may be tied. Critical values and powers for the modified Umbrella test (2009) are calculated using simulation technique under normal, exponential and logistic distributions. It is observed that power do not depend on the block effects and the test is very powerful.

Keywords: Modified umbrella test, Randomized block design, Tied observations, Ordered alternatives, Monte Carlo power study.

Introduction

Tied values occur when two or more observations are equal, whether the observations occur in the same sample or in different samples.There are some methods which solve the problem of tied observation for rank test. Methods are Midranks, Randomization, Average Statistic, Least Favorable Statistic, Average Probability, Range of Probabilityand Omission of Tied Observations.Earlier nonparametric work on Umbrella test by [3] was for the completely randomized design and later the modified Umbrella test introduced by [1] is applied for randomized block design (RBD) where tied values occur in treatments. For the Umbrella test, powers were not calculated.The motivation of this paper is to calculate approximate critical values and powers of the modified Umbrella test for different level of significance when there are ties in the observations for randomized block designs.Let Y_{ij} be a possibly unobserved score for block i and product j .Suppose we wish to compare t products or treatments and we have b blocks .Then the model is

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$$Y_{ij} = \mu + \beta_i + \tau_j + \varepsilon_{ij} \tag{1}$$

Where, μ is an overall effect, τ_j are treatment effects, β_i are block effects and ε_{ij} are independent and identically normally distributed random variables with mean zero.The following hypotheses are tested for the Umbrella *U* test (2009).

$$H_0: \tau_1 = \tau_2 = \ldots = \tau_t \; and$$
$$H_1 : \tau_1 \le \tau_2 \le \ldots \le \tau_p \ge \tau_{p+1} \ge \ldots \ge \tau_t$$
$$\text{or } H_2 : \tau_1 \ge \tau_2 \ge \ldots \ge \tau_p \le \tau_{p+1} \le \ldots \le \tau_t \; .$$

Here it is assumed that *p*is not known, and that at least one of the inequalities is strict. We refer to these as umbrella alternatives (because of the configuration of the corresponding population medians) and call "*p*" the point or peak of the umbrella. The peak of the Umbrella is unknown and is to estimate from the data. For tied observations the modified Umbrella statistic is definedby as

$$U = \kappa \left\{ M + \sum_{j=1}^{t} j(j-t-1)R_j \right\} \tag{2}$$

Where, $M = bt(t+2)(t+1)^2 /12$ and

$$\kappa^2 = \{180(t-1)\} / \{bptV(t^2-1)(t^2-4)\}$$

The variance of a rank is, $V = \left\{ \sum_{s=1}^{q} r_s^2 c_s /(bt) \right\} - (t+1)^2 / 4$

Where , q is the number of different values of s, b is the number of blocks , t is the number of products or treatments, p is the peak point, R_j is the sum of the ranks for the j^{th} product and r_s is the s^{th} ranking and c_s is the associated count.

In section 2,simulated approximate critical values are calculated from three probability distribution, Normal, Exponential and Logistic distribution, when the location of the peak is unknown. Section 3, presents the results of a Monte Carlo simulation investigation of the powers of the modified

Umbrella test considered in this paper for different levels of treatment and blocks. In section 4, a detailed example illustrating the use of these test procedure is given. We conclude in section 5 with some final remarks.

Approximate Critical Values

We have calculated the simulated critical values for the modified Umbrella test at 0.01, 0.05 and 0.10 significance level with various combinations of treatments and blocks. In this study we consider three underlying distributions such as Normal, Exponential and Logistic Distributions and 10000 replications are done to generate critical values for these distributions. Table 1 represents approximate critical values of different distributions for randomized block design (RBD).

Table 1: Simulated Critical Values for modified U Test Statistic under Normal (N), Exponential (E) and Logistic (L) Distributions.

Blocks b	Level	t = No. of treatments								
		3			4			5		
		N(θ_i,1)	E(θ_i,1)	L(θ_i,1)	N(θ_i,1)	E(θ_i,1)	L(θ_i,1)	N(θ_i,1)	E(θ_i,1)	L(θ_i,1)
3	0.01	3.2071	2.8284	3.1980	3.7032	3.5752	3.7712	4.2257	4.3787	4.9938
	0.05	1.8463	1.9364	1.9188	2.2478	2.2841	2.3570	2.5439	2.6561	3.0496
	0.10	1.4142	1.4142	1.3887	1.6499	1.5811	1.8194	1.8919	1.9917	2.2955
4	0.01	3.4016	3.4016	3.0618	3.3123	3.6742	3.9735	4.3320	4.2552	4.0011
	0.05	1.9364	1.9364	1.9639	2.1380	2.4161	2.3841	2.9505	2.7081	2.6845
	0.10	1.4173	1.3887	1.4173	1.5989	1.6813	1.7770	2.1702	1.9250	1.9153
5	0.01	3.1980	2.9199	3.0869	4.2008	4.1576	3.8890	4.3740	4.3465	4.4298
	0.05	1.7822	1.8371	1.8973	2.3841	2.3237	2.5010	2.5205	2.8301	2.7167
	0.10	1.4048	1.3258	1.4599	1.6903	1.9127	1.7864	1.9076	1.9477	1.9314
6	0.01	3.1658	2.9199	3.3203	3.3234	3.9727	4.0375	4.3515	4.6211	4.4523
	0.05	2.0380	1.8973	1.8708	2.1669	2.3533	2.6666	2.7167	3.0190	2.9951
	0.10	1.5434	1.3693	1.4048	1.5588	1.7597	1.9817	1.8410	1.9384	1.9794
7	0.01	3.2403	2.9199	2.9121	3.7635	4.1569	4.0441	4.4922	4.3569	4.3566
	0.05	2.1213	1.8973	1.9215	2.4333	2.4688	2.5041	2.7347	2.7704	2.9565
	0.10	1.4230	1.4048	1.3269	1.7260	1.7864	1.8229	2.0818	1.9876	1.9654
8	0.01	3.1658	3.3282	3.2071	3.9804	3.7635	3.7017	4.1096	4.5018	4.3972
	0.05	1.7693	1.9485	2.0044	2.3094	2.2322	2.3094	2.6684	2.9713	2.6579
	0.10	1.4142	1.4433	1.4560	1.7955	1.6514	1.6748	1.8898	1.9861	1.7888
9	0.01	3.2071	3.0310	3.0480	4.0698	3.8933	4.2146	4.6809	4.4932	4.8251
	0.05	2.0044	1.9364	1.9364	2.3590	2.5802	2.4647	3.1162	2.8013	2.9398
	0.10	1.4411	1.3568	1.4770	1.7084	1.8665	1.8036	2.1248	1.9250	2.1584
10	0.01	3.0480	2.8577	3.1819	3.4538	4.0329	3.8829	4.6670	4.0793	4.3568
	0.05	1.8190	1.7320	2.0647	2.1825	2.5123	2.3237	2.9518	2.5389	2.7713
	0.10	1.4142	1.2990	1.4342	1.5446	1.8330	1.7417	1.9597	1.7722	2.0673
11	0.01	3.2271	2.9698	3.3541	4.3414	3.9620	3.9589	4.3880	4.5241	4.1833
	0.05	1.8750	1.8631	1.9639	2.2597	2.2583	2.4373	2.8586	2.9885	2.5633
	0.10	1.3764	1.4342	1.4142	1.7591	1.6341	1.8073	2.0457	2.0505	1.8262
12	0.01	3.0186	3.1819	3.4874	3.9167	3.7269	3.8516	4.4752	4.1175	4.5534
	0.05	1.9877	1.6984	1.9409	2.4870	2.2983	2.4253	2.8229	2.5983	2.6827
	0.10	1.3764	1.2990	1.3587	1.6984	1.6514	1.7225	2.0673	1.9171	1.8530

From the above table we observed that the simulated critical values for modified Umbrella test statistic under different distributions show a decreasing pattern at 1%, 5% and 10% respectively at different levels of treatments and blocks.

Monte Carlo Power Study

The power of the rank test gives a measure for comparing different tests as well as for determining the sample sizes necessary to distinguish significant from a hypothesis with a reasonable degree of certainty. The power of the test procedure for detecting umbrella pattern with unique interior peak but varying spacing's between the populations medians are considered here. In particular, umbrella patterns in which there are different rates of ascent and descent of the population medians have been studied. Powers of the test statistics are worked out by choosing suitable critical values for different samples sizes. In this paper we present the results of a Monte Carlo power study for modified Umbrella (2009) U test for underlying three different distributions in Table 2 through 7.Although the various umbrella test procedures may detect monotone (i.e., ordered) patterns, the procedures are generally less powerful than those designed specifically to detect such patterns. To satisfy this purpose we calculate the last row in each Monte Carlo power study Table 2 through 7.

Table 2: Monte Carlo Power Estimates of modified umbrella U Test when $t = 4$ and $b = 10$ under Normal Distribution.

Design	Description	Umbrella Pattern	Location Parameters	Level	Power of U test
$t = 4$ $p = 3$ $b = 10$	$\lvert\theta_j - \theta_{j-1}\rvert = 0.5$	$\theta_1 < \theta_2 < \theta_3 > \theta_4$	$\theta_1 = 0,\ \theta_2 = 0.5,$ $\theta_3 = 1,\ \theta_4 = 0.5$	0.01 0.05 0.10	0.9903754 0.9499519 0.8999038
$t = 4$ $p = 2$ $b = 10$	$\theta_2 - \theta_1 = .5,$ $\theta_3 - \theta_2 = 1.5$	$\theta_1 < \theta_2 = \theta_3 > \theta_4$	$\theta_1 = 1,\ \theta_2 = 1.5,$ $\theta_3 = 1.5,\ \theta_4 = 0$	0.01 0.05 0.10	0.9901345 0.9497758 0.9004484
$t = 4$ $p = 3$ $b = 10$	$\theta_2 - \theta_1 = 0.5,$ $\theta_3 - \theta_2 = 1.5,$ $\theta_3 - \theta_4 = 1.75$	$\theta_1 < \theta_2 < \theta_3 > \theta_4$	$\theta_1 = 0,\ \theta_2 = 0.5,$ $\theta_3 = 2,\ \theta_4 = 0.25$	0.01 0.05 0.10	0.9900100 0.9520480 0.9010989

$t=4$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=.25,$ $\theta_3-\theta_4=.05$	$\theta_1<\theta_2<\theta_3>\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=.75,\ \theta_4=0.7$	0.01 0.05 0.10	0.9897770 0.9498141 0.8996283
$t=4$ $p=2$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_4=0.2$	$\theta_1<\theta_2=\theta_3>\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=0.5,\ \theta_4=0.3$	0.01 0.05 0.10	0.9909829 0.9504058 0.8999098
$t=4$ $p=4$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=0.1,$ $\theta_4-\theta_3=0.1$	$\theta_1<\theta_2<\theta_3<\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=0.6,\ \theta_4=0.7$	0.01 0.05 0.10	0.9900452 0.9502262 0.9004525

Comment: From the above table we observed the power study result for different umbrella pattern with unique interior peak and equally spaced medians on each side of the peak (but not equally- spaced throughout)show a decreasing pattern at 1%, 5% and 10% respectively. Simulations are based on Normal distribution with treatment is 4 and block is 10.

Table 3: Monte Carlo Power Estimates of modified umbrella U Test when $t=5$ and $b=10$ under Normal Distribution.

Design	Description	Umbrella Pattern	Location Parameters	Level	Power of U test
$t=5$ $p=3$ $b=10$	$\left\|\theta_j-\theta_{j-1}\right\|=0.5$	$\theta_1<\theta_2<\theta_3>\theta_4>\theta_5$	$\theta_1=0,\ \theta_2=0.5, \theta_3=1,$ $\theta_4=0.5, \theta_5=0$	0.01 0.05 0.10	0.9902534 0.9502924 0.8996101
$t=5$ $p=3$ $b=10$	$\theta_j-\theta_{j-1}=0.5$ for $j=2,3$ $\theta_{j-1}-\theta_j=1$ for $j=3,4$	$\theta_1<\theta_2<\theta_3>\theta_4>\theta_5$	$\theta_1=1,\ \theta_2=1.5, \theta_3=2,$ $\theta_4=1, \theta_5=0$	0.01 0.05 0.10	0.9902057 0.9500490 0.9000979
$t=5$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=1.5,$ $\theta_3-\theta_5=1.75$	$\theta_1<\theta_2<\theta_3=\theta_4>\theta_5$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=\theta_4=2, \theta_5=0.25$	0.01 0.05 0.10	0.9903288 0.9497099 0.9003868

$t=5$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=.25,$ $\theta_3-\theta_5=.05$	$\theta_1<\theta_2<\theta_3=\theta_4>\theta_5$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=\theta_4=0.75,\theta_5=0.7$	0.01 0.05 0.10	0.9896811 0.9502814 0.8996248
$t=5$ $p=4$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=0.3,$ $\theta_3-\theta_5=0.25$	$\theta_1<\theta_2=\theta_3<\theta_4>\theta_5$	$\theta_1=0,\ \theta_2=\theta_3=0.5,$ $\theta_4=0.8,\ \theta_5=0.6$	0.01 0.05 0.10	0.9896324 0.9500471 0.9000943
$t=5$ $p=5$ $b=10$	$\lvert\theta_j-\theta_{j-1}\rvert=0.75$	$\theta_1<\theta_2<\theta_3<\theta_4<\theta_5$	$\theta_1=0,\ \theta_2=0.75,$ $\theta_3=1.5,\ \theta_4=2.25,$ $\theta_5=3$	0.01 0.05 0.10	0.9901381 0.9497041 0.9003945
$t=5$ $p=5$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=0.1,$ $\theta_4-\theta_3=0.1$ $\theta_5-\theta_4=0.3$	$\theta_1<\theta_2<\theta_3<\theta_4<\theta_5$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=0.6,\ \theta_4=0.7,$ $\theta_5=1$	0.01 0.05 0.10	0.9895437 0.9496198 0.9001901

Comment: From the above table we observed the power study result for different umbrella pattern with unique interior peak and equally spaced medians on each side of the peak (but not equally- spaced throughout) show a decreasing pattern at 1%, 5% and 10% respectively. Simulations are based on Normal distribution with treatment is 5 and block is 10.

Table 4: Monte Carlo Power Estimates of modified umbrella U Test when $t=4$ and $b=10$ under Exponential Distribution.

Design	Description	Umbrella Pattern	Location Parameters	Level	Power of U test
$t=4$ $p=3$ $b=10$	$\lvert\theta_j-\theta_{j-1}\rvert=0.5$	$\theta_1<\theta_2<\theta_3>\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=1,\ \theta_4=0.5$	0.01 0.05 0.10	0.99100 0.95100 0.90200

$t=4$ $p=2$ $b=10$	$\theta_2-\theta_1=.5$, $\theta_3-\theta_2=1.5$	$\theta_1<\theta_2=\theta_3>\theta_4$	$\theta_1=1$, $\theta_2=1.5$, $\theta_3=1.5$, $\theta_4=0$	0.01 0.05 0.10	0.99100 0.95000 0.90200
$t=4$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5$, $\theta_3-\theta_2=1.5$, $\theta_3-\theta_4=1.75$	$\theta_1<\theta_2<\theta_3>\theta_4$	$\theta_1=0$, $\theta_2=0.5$, $\theta_3=2$, $\theta_4=0.25$	0.01 0.05 0.10	0.99050 0.95450 0.90500
$t=4$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5$, $\theta_3-\theta_2=.25$, $\theta_3-\theta_4=.05$	$\theta_1<\theta_2<\theta_3>\theta_4$	$\theta_1=0$, $\theta_2=0.5$, $\theta_3=.75$, $\theta_4=0.7$	0.01 0.05 0.10	0.99000 0.95200 0.90100
$t=4$ $p=2$ $b=10$	$\theta_2-\theta_1=0.5$, $\theta_3-\theta_4=0.2$	$\theta_1<\theta_2=\theta_3>\theta_4$	$\theta_1=0$, $\theta_2=0.5$, $\theta_3=0.5$, $\theta_4=0.3$	0.01 0.05 0.10	0.99150 0.95100 0.90600
$t=4$ $p=4$ $b=10$	$\theta_2-\theta_1=0.5$, $\theta_3-\theta_2=0.1$, $\theta_4-\theta_3=0.1$	$\theta_1<\theta_2<\theta_3<\theta_4$	$\theta_1=0$, $\theta_2=0.5$, $\theta_3=0.6$, $\theta_4=0.7$	0.01 0.05 0.10	0.99100 0.95000 0.90000

Comment: From the above table we observed the power study result for different umbrella pattern with unique interior peak and equally spaced medians on each side of the peak (but not equally- spaced throughout) show a decreasing pattern at 1%, 5% and 10% respectively. Simulations are based on Exponential distribution with treatment is 4 and block is 10.

Table 5: Monte Carlo Power Estimates of modified umbrella U Test when $t = 5$ and $b = 10$ under Exponential Distribution.

Design	Description	Umbrella Pattern	Location Parameters	Level	Power of U test
$t=5$ $p=3$ $b=10$	$\lvert\theta_j - \theta_{j-1}\rvert = 0.5$	$\theta_1 < \theta_2 < \theta_3 > \theta_4 > \theta_5$	$\theta_1 = 0,\ \theta_2 = 0.5, \theta_3 = 1,$ $\theta_4 = 0.5, \theta_5 = 0$	0.01 0.05 0.10	0.99000 0.95100 0.90000
$t=5$ $p=3$ $b=10$	$\theta_j - \theta_{j-1} = 0.5$ for $j = 2,3$ $\theta_{j-1} - \theta_j = 1$ for $j = 3,4$	$\theta_1 < \theta_2 < \theta_3 > \theta_4 > \theta_5$	$\theta_1 = 1,\ \theta_2 = 1.5, \theta_3 = 2,$ $\theta_4 = 1, \theta_5 = 0$	0.01 0.05 0.10	0.99000 0.95000 0.90100
$t=5$ $p=3$ $b=10$	$\theta_2 - \theta_1 = 0.5,$ $\theta_3 - \theta_2 = 1.5,$ $\theta_3 - \theta_5 = 1.75$	$\theta_1 < \theta_2 < \theta_3 = \theta_4 > \theta_5$	$\theta_1 = 0,\ \theta_2 = 0.5,$ $\theta_3 = \theta_4 = 2, \theta_5 = 0.25$	0.01 0.05 0.10	0.99100 0.95000 0.90000
$t=5$ $p=3$ $b=10$	$\theta_2 - \theta_1 = 0.5,$ $\theta_3 - \theta_2 = .25,$ $\theta_3 - \theta_5 = .05$	$\theta_1 < \theta_2 < \theta_3 = \theta_4 > \theta_5$	$\theta_1 = 0,\ \theta_2 = 0.5,$ $\theta_3 = \theta_4 = 0.75, \theta_5 = 0.7$	0.01 0.05 0.10	0.99000 0.95000 0.90000
$t=5$ $p=4$ $b=10$	$\theta_2 - \theta_1 = 0.5,$ $\theta_3 - \theta_2 = 0.3,$ $\theta_3 - \theta_5 = 0.25$	$\theta_1 < \theta_2 = \theta_3 < \theta_4 > \theta_5$	$\theta_1 = 0,\ \theta_2 = \theta_3 = 0.5,$ $\theta_4 = 0.8,\ \theta_5 = 0.6$	0.01 0.05 0.10	0.99000 0.95000 0.90000
$t=5$ $p=5$ $b=10$	$\lvert\theta_j - \theta_{j-1}\rvert = 0.75$	$\theta_1 < \theta_2 < \theta_3 < \theta_4 < \theta_5$	$\theta_1 = 0,\ \theta_2 = 0.75,$ $\theta_3 = 1.5,\ \theta_4 = 2.25,$ $\theta_5 = 3$	0.01 0.05 0.10	0.99000 0.95200 0.90000

$t=5$ $p=5$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=0.1,$ $\theta_4-\theta_3=0.1$ $\theta_5-\theta_4=0.3$	$\theta_1<\theta_2<\theta_3<\theta_4<\theta_5$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=0.6,\ \theta_4=0.7,$ $\theta_5=1$	0.01 0.05 0.10	0.99000 0.95000 0.90100

Comment: From the above table we observed the power study result for different umbrella pattern with unique interior peak and equally spaced medians on each side of the peak (but not equally- spaced throughout) show a decreasing pattern at 1%, 5% and 10% respectively. Simulations are based on Exponential distribution with treatment is 5and block is 10.

Table 6: Monte Carlo Power Estimates of modified umbrella U Test when $t=4$ and $b=10$ under Logistic Distribution.

Design	Description	Umbrella Pattern	Location Parameters	Level	Power of U test
$t=4$ $p=3$ $b=10$	$\lvert\theta_j-\theta_{j-1}\rvert=0.5$	$\theta_1<\theta_2<\theta_3>\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=1,\ \theta_4=0.5$	0.01 0.05 0.10	0.9910233 0.9497307 0.9003591
$t=4$ $p=2$ $b=10$	$\theta_2-\theta_1=.5,$ $\theta_3-\theta_2=1.5$	$\theta_1<\theta_2=\theta_3>\theta_4$	$\theta_1=1,\ \theta_2=1.5,$ $\theta_3=1.5,\ \theta_4=0$	0.01 0.05 0.10	0.9942912 0.9495718 0.9010466
$t=4$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=1.5,$ $\theta_3-\theta_4=1.75$	$\theta_1<\theta_2<\theta_3>\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=2,\ \theta_4=0.25$	0.01 0.05 0.10	0.9902534 0.9502924 0.9005848
$t=4$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=.25,$ $\theta_3-\theta_4=.05$	$\theta_1<\theta_2<\theta_3>\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=.75,\ \theta_4=0.7$	0.01 0.05 0.10	0.9902222 0.9502222 0.8995556

$t=4$ $p=2$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_4=0.2$	$\theta_1<\theta_2=\theta_3>\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=0.5,\ \theta_4=0.3$	0.01 0.05 0.10	0.9896462 0.9499569 0.8999137
$t=4$ $p=4$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=0.1,$ $\theta_4-\theta_3=0.1$	$\theta_1<\theta_2<\theta_3<\theta_4$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=0.6,\ \theta_4=0.7$	0.01 0.05 0.10	0.9902396 0.9503106 0.8997338

Comment: From the above table we observed the power study result for different umbrella pattern with unique interior peak and equally spaced medians on each side of the peak (but not equally- spaced throughout) show a decreasing pattern at 1%, 5% and 10% respectively. Simulations are based on Logistic distribution with treatment is 4and block is 10.

Table 7: Monte Carlo Power Estimates of modified umbrella U Test when $t=5$ and $b=10$ under Logistic Distribution.

Design	Description	Umbrella Pattern	Location Parameters	Level	Power of U test
$t=5$ $p=3$ $b=10$	$\lvert\theta_j-\theta_{j-1}\rvert=0.5$	$\theta_1<\theta_2<\theta_3>\theta_4>\theta_5$	$\theta_1=0,\ \theta_2=0.5, \theta_3=1,$ $\theta_4=0.5, \theta_5=0$	0.01 0.05 0.10	0.9895338 0.9495718 0.9000951
$t=5$ $p=3$ $b=10$	$\theta_j-\theta_{j-1}=0.5$ for $j=2,3$ $\theta_{j-1}-\theta_j=1$ for $j=3,4$	$\theta_1<\theta_2<\theta_3>\theta_4>\theta_5$	$\theta_1=1,\ \theta_2=1.5, \theta_3=2,$ $\theta_4=1, \theta_5=0$	0.01 0.05 0.10	0.9914286 0.9504762 0.9000000
$t=5$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=1.5,$ $\theta_3-\theta_5=1.75$	$\theta_1<\theta_2<\theta_3=\theta_4>\theta_5$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=\theta_4=2, \theta_5=0.25$	0.01 0.05 0.10	0.9903475 0.9498069 0.8996139

$t=5$ $p=3$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=.25,$ $\theta_3-\theta_5=.05$	$\theta_1<\theta_2<\theta_3=\theta_4>\theta_5$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=\theta_4=0.75, \theta_5=0.7$	0.01 0.05 0.10	0.9904489 0.9503343 0.8997135
$t=5$ $p=4$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=0.3,$ $\theta_3-\theta_5=0.25$	$\theta_1<\theta_2=\theta_3<\theta_4>\theta_5$	$\theta_1=0,\ \theta_2=\theta_3=0.5,$ $\theta_4=0.8,\ \theta_5=0.6$	0.01 0.05 0.10	0.9896422 0.9500942 0.9001883
$t=5$ $p=5$ $b=10$	$\lvert\theta_j-\theta_{j-1}\rvert=0.75$	$\theta_1<\theta_2<\theta_3<\theta_4<\theta_5$	$\theta_1=0,\ \theta_2=0.75,$ $\theta_3=1.5,\ \theta_4=2.25,$ $\theta_5=3$	0.01 0.05 0.10	0.9903288 0.9497099 0.9003868
$t=5$ $p=5$ $b=10$	$\theta_2-\theta_1=0.5,$ $\theta_3-\theta_2=0.1,$ $\theta_4-\theta_3=0.1$ $\theta_5-\theta_4=0.3$	$\theta_1<\theta_2<\theta_3<\theta_4<\theta_5$	$\theta_1=0,\ \theta_2=0.5,$ $\theta_3=0.6,\ \theta_4=0.7,$ $\theta_5=1$	0.01 0.05 0.10	0.9895338 0.9495718 0.9000951

Comment: From the above table we observed the power study result for different umbrella pattern with unique interior peak and equally spaced medians on each side of the peak (but not equally- spaced throughout) show a decreasing pattern at 1%, 5% and 10% respectively. Simulations are based on Logistic distribution with treatment is 5 and block is 10.

An Empirical Example

Let us consider the partially modified data given in the following table corresponding to an experimental design discussed in [2], which yielded data exhibiting a dose-response relationship with a downturn in response at high doses. Five male albino rats of the same strain and of approximately the same weight were utilized. Four dosage levels of the drug were studied.

Drug effect data

Rat	Dosage (mg/kg)			
	0.5	1.0	1.5	2.0
1	0.81(3)	0.80(2)	0.60(1)	0.82(4)
2	0.78(3)	0.61(2)	0.51(1)	0.79(4)
3	0.80(2)	0.82(3)	0.62(1)	0.83(4)
4	0.95(3.5)	0.95(3.5)	0.60(1)	0.91(2)
5	1.13(4)	0.82(1)	0.92(2)	1.04(3)
R_j	15.5	11.5	6	17

Here, dosage levels are the treatments and rats are the block effects.
So, treatment $t=4$ and block $b=5$, $p=3$

Then counts of each ranking are given below:

Ranks (r_s)	1	2	3	3.5	4
Count (c_s)	5	5	4	2	4

From the above data we find $U=3.03$ and approximate critical value for U is 2.38 (from Table 1).So, it appears that there is significant treatment effect at the 2.0 mg/kg dosage. It appears there is an umbrella effect at high doses 2.0 mg/kg.

Conclusion

In this paper, we have proposed modified umbrella test for umbrella alternative in balanced randomized block design.This test identified the probable location of an umbrella peak the data based.The (advantage) superiority of this test is measured by Monte Carlo Simulation power study to detect umbrella pattern along with monotone pattern when peak is unknown.Finally we can be concluding that, it allows the researcher to apply this test for RBD in the tied observations.

References

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[3] Mack, G. A. and Wolfe, D. G. (1981): K-Sample Rank Tests for Umbrella Alternatives, *Journal of the American Statistical Association.*,Vol. **76**, pp 175-181.